Precalc notes 10.1

Intro to limits

The concept of the limit is fundamental to the study of calculus.

1) Find the maximum area for a rectangle that has a perimeter of 28.

If f (x) becomes arbitrarily close to a unique number L
as x approaches c from either side, then

$$\lim_{x \to c} f(x) = L$$

Basic Limits

Let b and c be real numbers and let n be a positive integer.

1. $\lim_{x \to c} b = b$
2. $\lim_{x \to c} x = c$
3. $\lim_{x \to c} x^n = c^n$
4. $\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$, valid for all c when n is odd and valid for $c > 0$ when n is even

Properties of Limits

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = K$$

1. Scalar multiple: $\lim_{x \to c} [b f(x)] = bL$
2. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
3. Product: $\lim_{x \to c} [f(x)g(x)] = LK$
4. Quotient: $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$, provided $K \neq 0$
5. Power: $\lim_{x \to c} [f(x)]^n = L^n$

Use a table to find the limit.

2) $\lim_{x \to 3} \frac{x+2}{\sqrt{x+1}}$

3) $\lim_{x \to 5} \frac{x^2 - 7x + 10}{x - 5}$

4) Find $\lim_{x\to 8} f(x)$ for $f(x)=\begin{cases}4x-7, x\neq 8\\ 12, x=8\end{cases}$

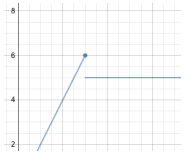
5) $\lim_{x\to c} f(x)=5$ $\lim_{x\to c} g(x)=-2$

a. $\lim_{x\to c}\left[2f(x)\right]$ b. $\lim_{x\to c}\dfrac{f(x)}{g(x)}$ c. $\lim_{x\to c}\left[\sqrt{5f(x)}+2g(x)\right]$

When do limits not exist?

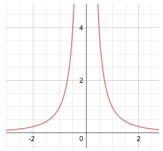
1. When left and right behaviors are different

$$\lim_{x\to 3} f(x) \quad \text{for} \quad f(x)=\begin{cases}2x, x\leq 3\\ 5, x>3\end{cases}$$



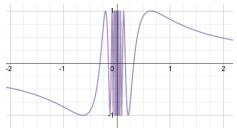
2. Unbounded behavior

$$\lim_{x\to 0}\frac{1}{x^2}$$



3. Oscillating behavior

$$\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$$



Use substitution to find the limit.

a) $\lim_{x\to 3} x^3$ b) $\lim_{x\to -2} 4x$

c) $\lim_{x\to \pi}\dfrac{\cos x}{3x}$ d) $\lim_{x\to 25}\left(2x+\sqrt{x}\right)$

Use substitution to find the limit.

e) $\lim_{x\to \frac{\pi}{2}}\left(\cot x\sin x\right)$ f) $\lim_{x\to 3}\left(2-x\right)^2$

g) $\lim_{x\to -2}\left(x^2+7x+12\right)$ h) $\lim_{x\to -2}\left(\dfrac{x^2+7x+12}{x+4}\right)$